# Refinements in Crystal Ladder Filter Design

*Improved design techniques can result in much better wide-bandwidth filters.*

By Wes Hayward, W7ZOI

Hams have been building their own crystal filters since the earliest days of single sideband. Early motivations were economic; commercially built filters were either too expensive or unavailable. Quartz crystals offered a method for achieving the selectivity needed in SSB transmitters and receivers. The trend continues, especially among QRP enthusiasts.

Recent work by Carver and by Makhinson has taken the process further.[1,2] Both examined the construction and design of very high-performance crystal filters. Their goal was to build filters offering performance that was not commercially available. This work produced some spectacular filter performance. Both based their work on one of several versions of *X.EXE*, a computer program for crystal filter design that I wrote several years ago.[3]

Recent communications (including those with Makhinson) mention limitations in the design method used in *X.EXE*. Wide filters were difficult, if not impossible. Examination of the underlying filter theory revealed a simple *circuit modification* to extend the design methods. This circuit extension produces filters with wider bandwidths at a greater variety of center frequencies, including filters using overtone modes. The new freedom allows us to build filters with improved time-domain performance, the primary goal in the filter designs of Carver.

Before examining circuit modifications, the original mathematical methods will be presented—allowing the limitations to be evaluated—beginning with a review of L-C filters using crystal-like circuits. The problems encountered when we substitute crystals into this framework are then presented, leading to the methods used in *X.EXE*. Finally, the circuit modifications are presented that extend the capabilities.

This article is practical to the extent that it produces filters that are inexpensive and relatively easy to build. The filters really work well. The methods, however, are mathematical. Hence, the article is more analytical than is usual in amateur literature.

[1]Notes appear on page 21.

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### L-C Filter Background

The bandwidth of a band-pass filter defines a filter Q. This parameter, $Q_f = F_{center} / Bandwidth$, must be less than the unloaded Q of the resonators. A filter is then completely determined if the following conditions are met:

1) The singly loaded end section Q is

established in accordance with the polynomial of choice (Chebyshev, Bessel, etc).

2) The couplings between resonators are set to fit the polynomial.

A practical, although uncommon, example would use L-C *series* tuned circuits with large L and small C. This topology is presented in Fig 1.[4]

Unloaded resonator Q is easily measured. Once known, a *normalized* Q can be calculated for a filter as the ratio, $Q_0 = Q_u / Q_f$. Tables in Zverev list the values of $Q_0$ that are needed for a given polynomial filter to be realizable.[5] If the resonators are not good enough, it will be impossible to realize some filter types.

Consider an example, a 4th-order Butterworth filter, which has normalized parameters $q_1 = q_4 = 0.765$, $k_{12} = 0.841$, $k_{23} = 0.541$, and $k_{34} = 0.841$. Assume that the inductors are 10 µH while the series capacitor is 101.3 pF. Assume also that both L and C are fixed; we can't adjust them. (This restriction is the same one we encounter with crystals.) The L-C combination is series resonant at 5 MHz. Assume further that $Q_u = 250$, a typical value for toroid inductors. Examination of Zverev's tables shows that $Q_0 > 3.7$ will allow the construction of a 4-resonator Butterworth filter. The unloaded resonator bandwidth is 20 kHz, so 4th-order Butterworth band-pass filters with a bandwidth of 74 kHz or more are realizable. We will design this filter for $B = 200$ kHz, so $Q_f = 25$ at 5 MHz.

The end section Q is denormalized according to the equations summarized on page 92 of Note 3:

$$Q_{END} = \left( \frac{1}{q \cdot Q_f} - \frac{1}{Q_u} \right)^{-1} \qquad \text{Eq 1}$$

where $q$ (lower case) is the normalized end Q. The value for $q$ was 0.765 for both ends of the Butterworth filter, so denormalized $Q_{END} = 20.71$. The inductor reactance at the filter center frequency is $\omega L = 314.2\ \Omega$, so the end sections will have a singly loaded Q of 20.71 if the end termination is 15.2 Ω. The coupling coefficients are denormalized with regard to $Q_f$:

$$K_{12} = \frac{k_{12}}{Q_f} \qquad \text{Eq 2}$$

where $k_{12}$ (lower case) is the normalized coupling coefficient between the 1st and 2nd resonator. The coupling capacitor is then:

$$C_{12} = \frac{C_0}{K_{12}} \qquad \text{Eq 3}$$

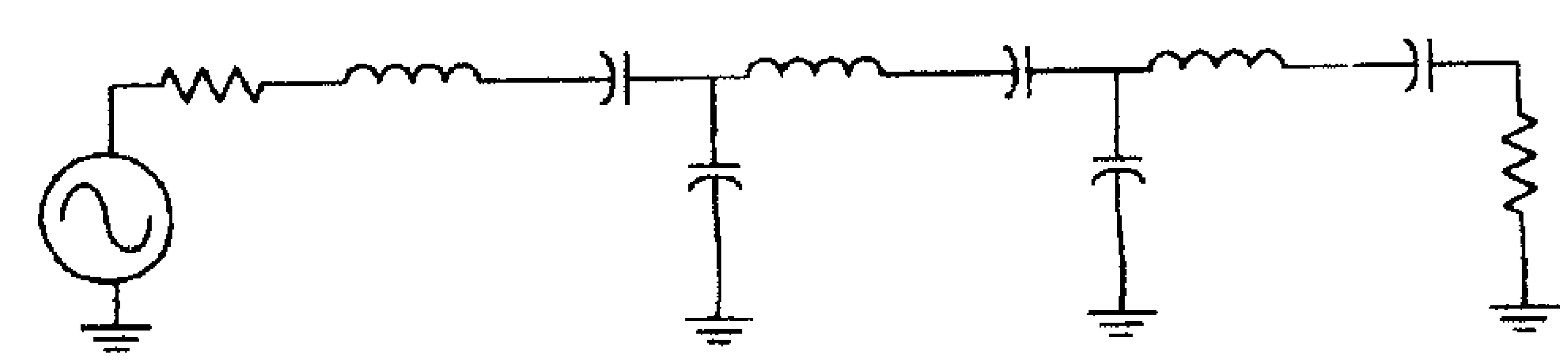

Fig 1—L-C filter topology using series tuned circuits.

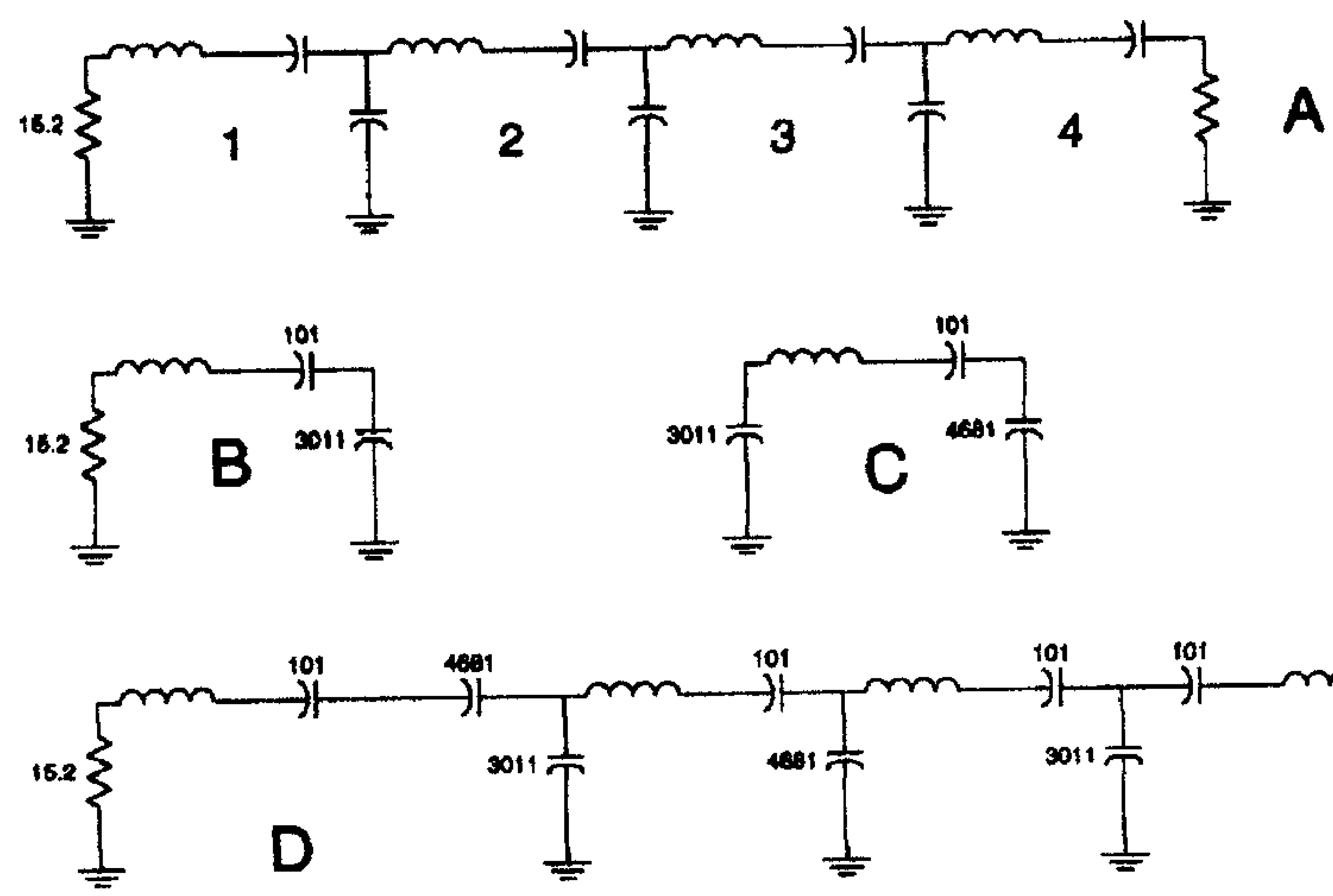


Fig 2—Evolution of a 4-resonator band-pass L-C filter including tuning. See text.

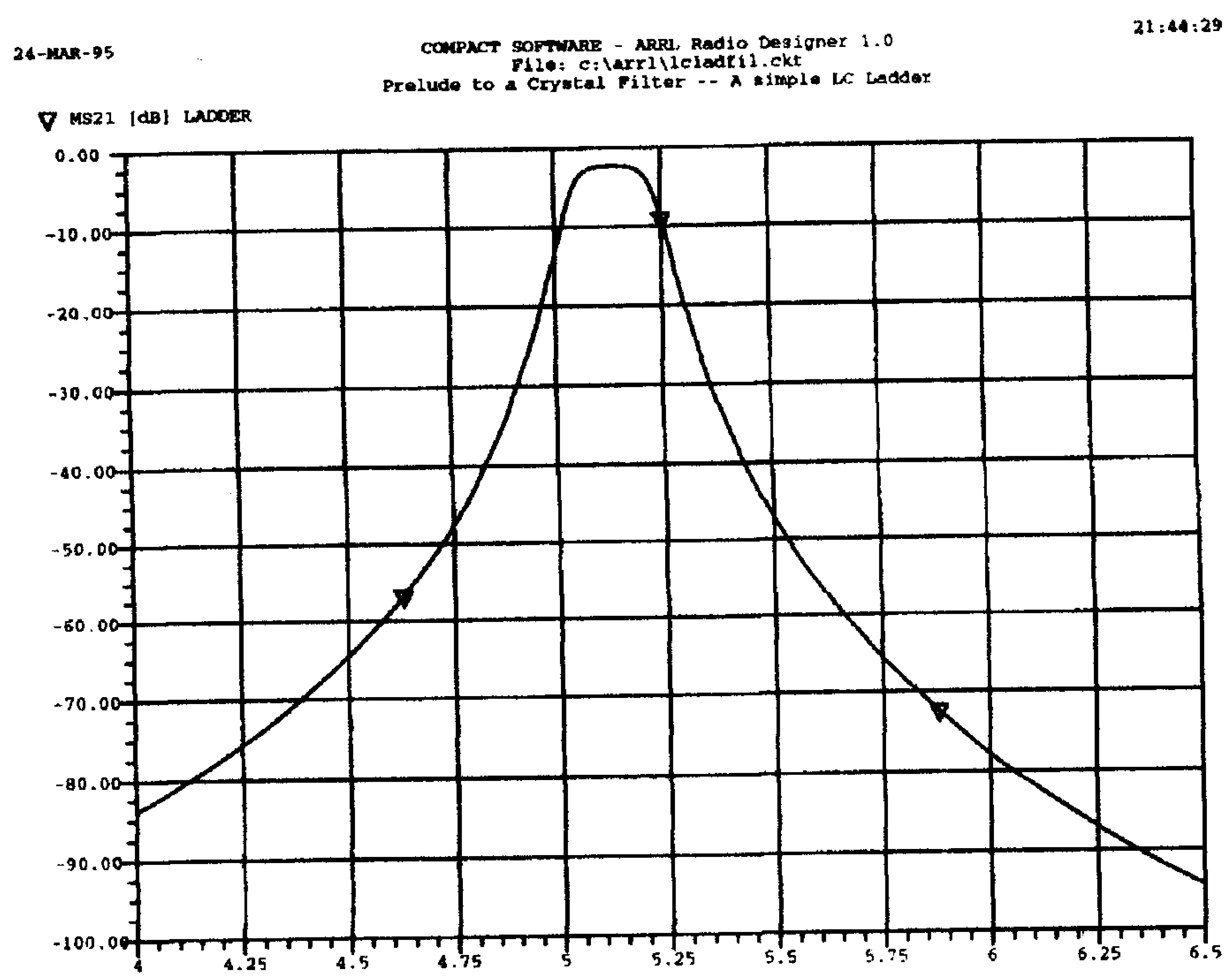


Fig 3—L-C filter response. The filter is that of Fig 2D.

where $C_o$ is the nodal capacitance for the resonator, 101.3 pF for the example. With $k_{12} = 0.841$, the coupling capacitor is $C_{12} = 3011$ pF, a large but realizable value. $C_{34}$ has the same value. A similar process generates the middle coupling capacitor, $C_{23} = 4681$ pF.

The almost-complete filter is shown in Fig 2A. We must still tune the filter. The simple resonators all started on the same 5-MHz frequency, but this synchronous tuning was upset when we inserted coupling capacitors. Mesh 1, shown in Fig 2B, is resonant at 5.084 MHz while mesh 2 (Fig 2C) tunes to 5.137 MHz. Owing to symmetry, mesh 3 is identical to mesh 2 while mesh 4 is the same as mesh 1. A filter becomes properly tuned when we insert extra series C in meshes 1 and 4, forcing all four meshes to be resonant at the same frequency, 5.137 MHz. The final circuit is shown in Fig 2D. The calculated response of this filter is presented in Fig 3.

### The Quartz Crystal

The equivalent circuit for a crystal is given in Fig 4. The motional parameters, $L_m$ and $C_m$, form a series tuned circuit while ESR, the *equivalent series resistance*, models crystal losses. This is, so far, no different from the series tuned circuits considered in the previous L-C example. However, the quartz crystal is complicated by an additional element, $C_o$, the parallel crystal capacitance. This component is the result of the basic physical crystal structure, a slab of quartz with metalization on both sides. This is a simple parallel-plate capacitor that is unrelated to the piezoelectric properties of the material. $C_o$ of Fig 4 includes any stray capacitance that might be in the crystal package. The value for $C_o$ is usually related to the motional capacitance by an approximation, $C_o = 220C_m$.[6]

The component values are much different than what we might realize with off-the-shelf L and C. For example, a 10-MHz crystal that I've used for numerous filter experiments has $L_m = 0.02$ H, $C_m = 12.67$ fF, and $C_o = 3$ pF. (1 fF = 0.001 pF = 1 femtofarad.)

As an initial approximation, the filter designer might ignore $C_o$. The crystal is then nothing more than a series resonator, and the design of filters is *exactly* like the L-C example. Unfortunately, this works only for the narrowest of filters. As the filter becomes wider, the discrepancy between design and measured results is extreme. (It is not necessary to build the filters to see these effects. The difficulties are quite evident in computer simulation with $C_o$ included in the crystal model.) The coupling capacitors calculated when $C_o$ is ignored are too large and may not be in the right ratio. The result is a bandwidth that is too narrow and a distorted shape. A smaller value of motional C should be used when calculating coupling elements. But what are appropriate motional elements? An answer was found in a *reactance slope approximation*, the method that formed the basis for *X.EXE*.

Consider a simple lossless series tuned circuit, Fig 5. The complex impedance is given as:

$$Z = j \cdot X = j\left(\omega \cdot L - \frac{1}{\omega \cdot C}\right) \quad \text{Eq 4}$$

where $\omega = 2\pi f$. At resonance:

$$\omega_o \cdot C = \frac{1}{\omega_o \cdot L} \quad \text{Eq 5}$$

which allows us to eliminate $C$:

$$j \cdot X(\omega) = j\left(\omega \cdot L - \frac{\omega_o^2 \cdot L}{\omega}\right) \quad \text{Eq 6}$$

Differentiating this expression with respect to angular frequency, $\omega$:

$$\frac{\partial X}{\partial \omega} = L + \frac{\omega_o^2 \cdot L}{\omega^2} \quad \text{Eq 7}$$

This expression can be solved for inductance as a function of reactance slope with frequency, providing a new definition for effective inductance:

$$L = \frac{\frac{\partial X}{\partial \omega}}{1 + \frac{\omega_o^2}{\omega^2}} \quad \text{Eq 8}$$

Consider now the equivalent circuit for the crystal shown in Fig 4. Ignoring ESR, the series tuned circuit portion has the impedance:

$$Z_{SER} = j\left(\omega \cdot L - \frac{\omega_o^2 \cdot L}{\omega}\right) \quad \text{Eq 9}$$

This is converted to an admittance, $Y = 1/Z$, and the admittance of the parallel capacitance, $C_o$, is added, yielding:

$$Y = j\left[\frac{\omega \cdot C_o \cdot L \cdot (\omega^2 - \omega_o^2) - \omega}{L \cdot (\omega^2 - \omega_o^2)}\right] \quad \text{Eq 10}$$

Converting back to impedance form, the reactance becomes

$$X = \frac{L_m \cdot (\omega_o^2 - \omega^2)}{\omega \cdot C_o \cdot L_m \cdot (\omega^2 - \omega_o^2) - \omega} \quad \text{Eq 11}$$

This reactance is differentiated with respect to angular frequency, $\omega$, with the result inserted into Eq 8 to produce the desired result, an expression for effective resonator inductance of a crystal in a filter:

$$L_{EFF} = L_m \cdot \left[\frac{2 \cdot A}{(A-1)^2 \cdot \left(1 + \frac{\omega_o^2}{\omega^2}\right)} - \frac{1}{A-1}\right] \quad \text{Eq 12}$$

where

$$A = L_m \cdot C_o \cdot (\omega^2 - \omega_o^2) \quad \text{Eq 13}$$

We define $X(\delta f)$ as the ratio of the effective inductance to the motional inductance, with $\delta f$ being an offset frequency in Hertz above the crystal resonance. Then, Fig 6 shows $X(\delta f)$ plotted as a function of offset for a typical 5-MHz crystal with $L_m = 0.098$ H. Two parallel capacitance values are used, resulting in the two curves. The lower curve is for $C_o = 2$ pF while the upper one is for $C_o = 5$ pF. The effective inductance ratio is plotted for offsets up to 2500 Hz. The effect is dramatic, especially when $C_o$ is large.

This effect, an increase in effective inductance resulting from added parallel capacitance, is a familiar one. It happens when we parallel an inductor in a low-pass filter to generate fre-

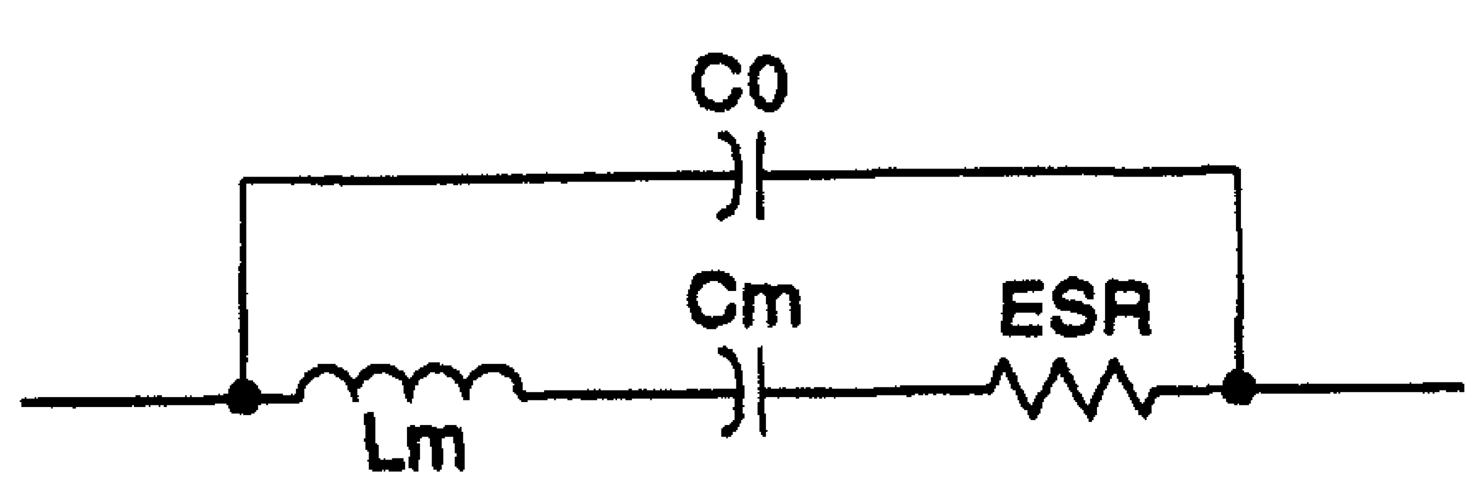


**Fig 4—Quartz crystal model.**

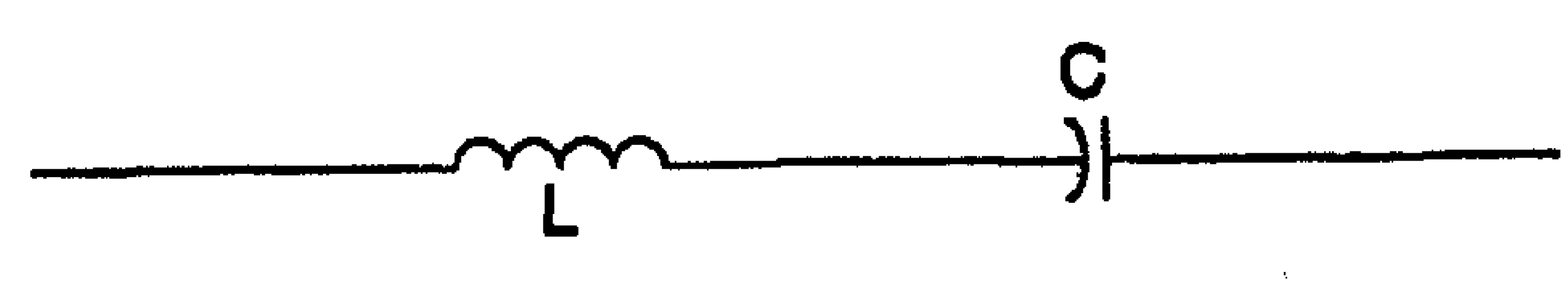


**Fig 5—Ideal series tuned circuit.**



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quencies of high attenuation in the stopband (elliptic filters). It also happens when we build a trap for use in an antenna. In both cases, the addition of parallel C causes the inductor to behave like one with a larger value.

### Crystal Filter Design

Now that we have isolated an effect—increased effective inductance due to holder capacitance—that will complicate filter performance, we can try to compensate for it. From Fig 6, if the filter is narrow, with a bandwidth of only a few hundred Hertz, the inductance ratio is very close to 1 over the entire band. We can then build effective filters by allowing the inductance to equal the motional value. Wider-bandwidth filters are built by evaluating the effective L over the passband. The $L_{eff}$ value calculated at the top of the passband seems to work well for SSB filters. That value, and the resulting motional capacitances, are used for filter design. This is done in *X.EXE*.

The results shown in Fig 6 describe a particular 5-MHz crystal, although the effect is a general one. Going to a higher crystal frequency partially alleviates the problem, while lower frequencies complicate the outcome. This is the effect that led Makhinson (Note 2) to observe that effective lower-sideband ladder filters with an SSB bandwidth are best built for center frequencies between 6 and 12 MHz.

The approach used in *X.EXE* seems to produce useful and predictable filters when $L_{eff} < 2L_m$. The rest of the design is no different than the L-C method outlined above. The reader should review the results obtained by Carver and Makhinson to see what is possible with these methods.

Motional inductance and capacitance can be measured with a variety of methods. The scheme shown in the sidebar is the one I presently use with most crystals.

### Additional Refinements

The methods presented are not new. Rather, they represent but one mathematical formulation for the design. This method grew from correspondence with Dr. Dave Gordon-Smith, G3UUR. Dave had used similar analytical techniques to derive a unique set of normalized filter tables for crystal filter design with arbitrary crystals with any value for C0.[7] While not published, Dave has freely distributed his work (and tables) to many amateur experimenters over the years. His analysis predated and contributed to the "$L_m$ effective" approach that I've presented above (Eq 12). Dave also originated the crystal measurement scheme presented in the sidebar.

### Wider Bandwidth Crystal Filters

The design methods outlined so far do not work well (if at all) when wide-bandwidth filters are to be built. Even when the methods appear to be working with regard to filter shape and bandwidth, the resulting circuits may have excessive group delay near one end of the passband. This is another consequence of $C_o$, the nemesis of our problem. There is a simple solution to the problem: add circuitry that will cancel the effects of $C_o$.

The modified design procedure will be illustrated with an example. The circuit desired is one with a bandwidth of 3 kHz, suitable for AM reception. The center frequency is 3.58 MHz, the frequency of readily available color-

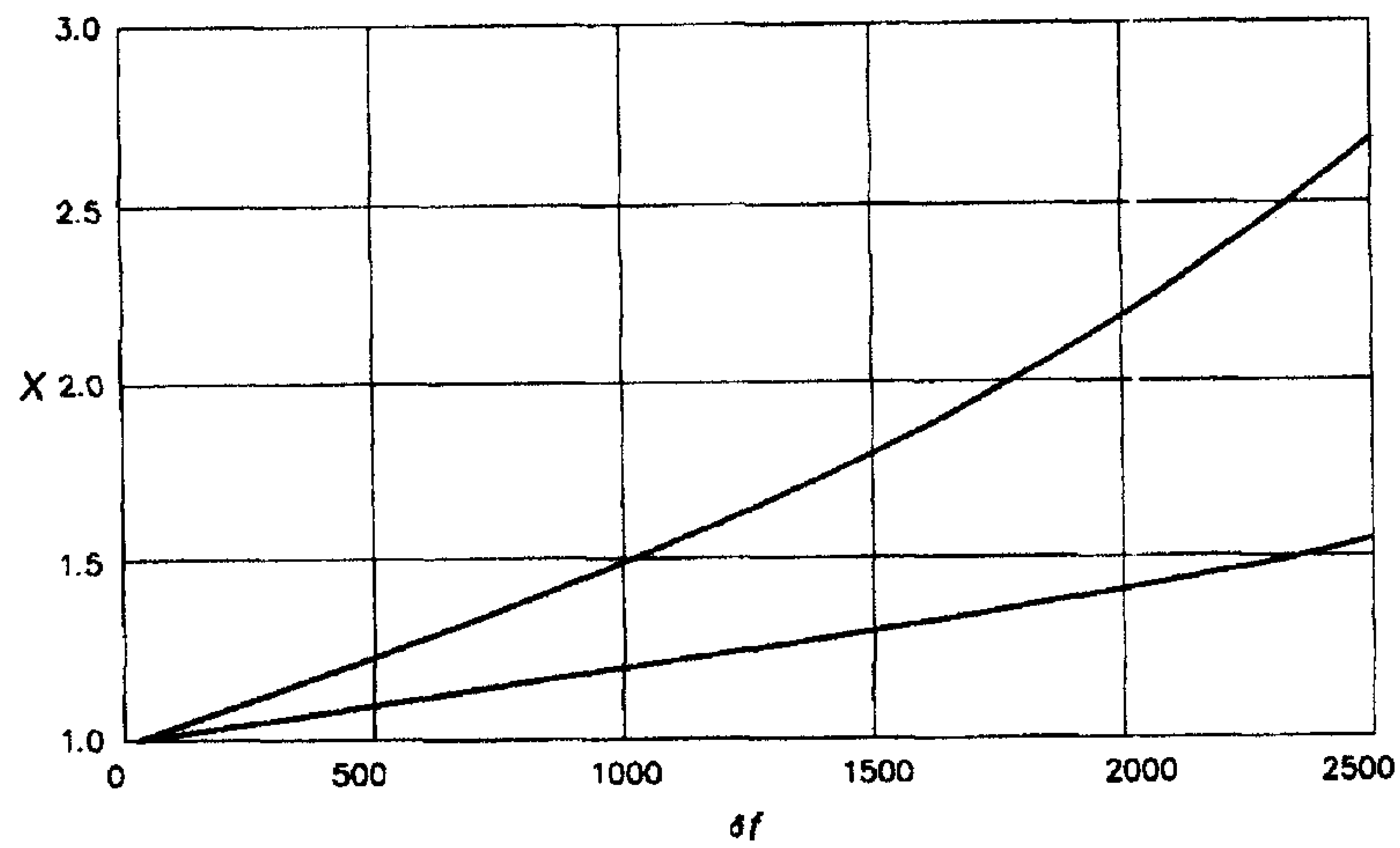



Fig 6—*X*, defined as $L_{eff}/L_m$, is plotted for frequency offset, $\delta f$, above the crystal series-resonance frequency in Hertz. These 5-MHz crystals had parallel C of 2 pF (lower curve) and 5 pF.

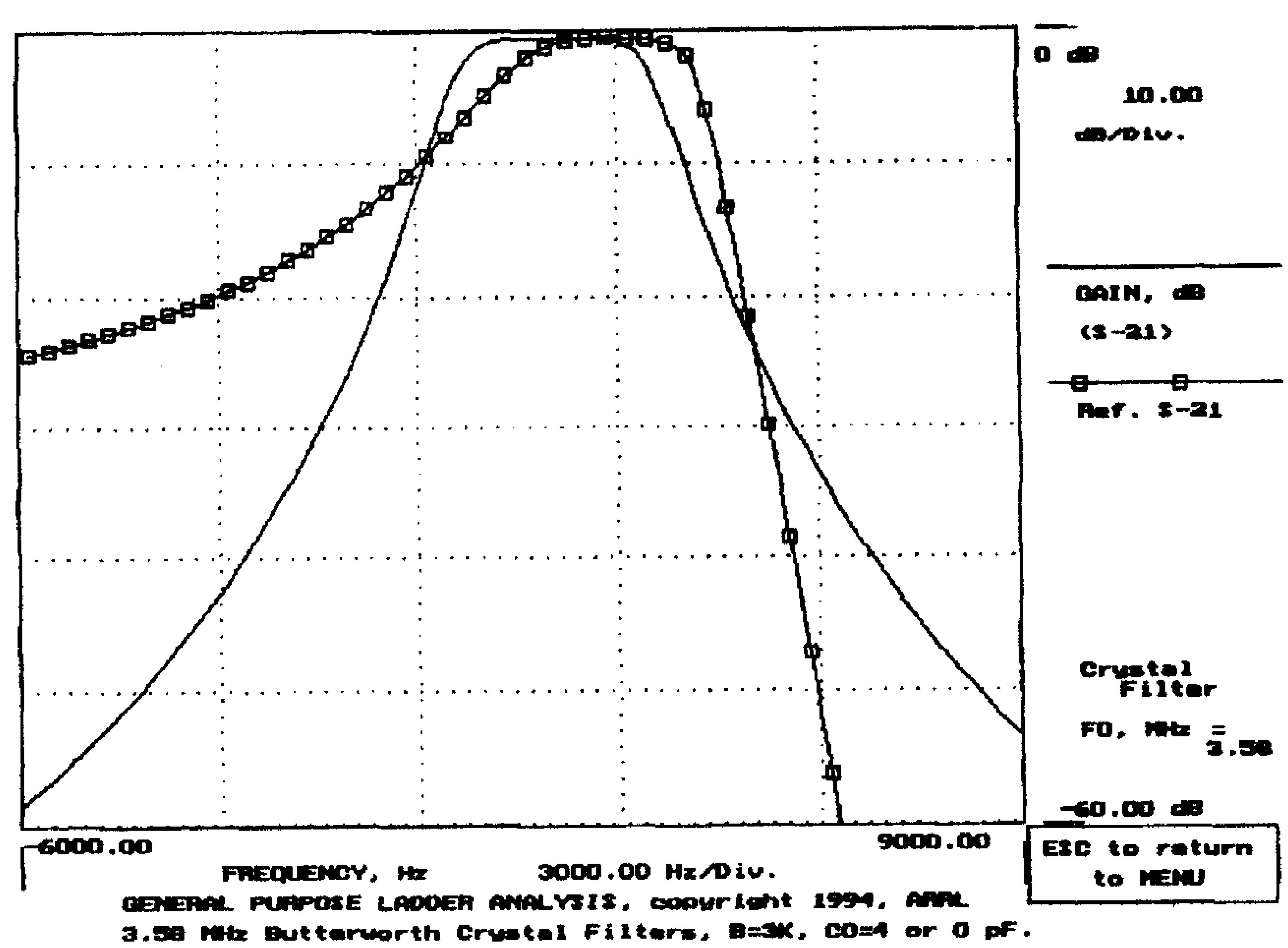



Fig 7

burst crystals. Circuit design begins with an attempt at using the standard methods. The available crystals have a motional L of 0.117 H and $C_0$ of 4 pF. This data, along with the $k$ and $q$ data for a Butterworth polynomial were entered into *X.EXE*. The resulting filter was then analyzed; the results are presented in the *reference plot* (small squares) of Fig 7. The filter is a poor one at best, showing severe asymmetry. Further, the bandwidth is slightly less than the desired 3 kHz. This attempt at synthesis is clearly unsuccessful; this is an application requiring the modified circuit.

The scheme that is used to cancel $C_0$, the parallel crystal C, is to add inductance in parallel with the crystal, creating a parallel resonance with $C_0$. The inductance is made a bit smaller than needed and a small trimmer capacitance is added, allowing for easy adjustment. The resulting filter is shown in Fig 8. The filter was designed with *X.EXE*, but with a value of 0 pF entered for $C_0$.

The added inductors are wound on ferrite toroids. This was required for this low frequency filter; the needed 150-µH inductance was not practical with iron-powder toroids. The solid curve in Fig 7 shows the calculated result for this filter. The shape is clean and very symmetrical, something of a rarity with lower-sideband-ladder crystal circuits.

This filter was built and tested with the results shown in Fig 9. The only subtlety encountered was with adjustment of the trimmer capacitors. I eventually shorted all four crystals with pieces of wire. Then the crystals were individually unshorted, the related capacitor was adjusted for a deep null about 20 kHz away from the passband, and the crystal was again short circuited. Only then were the shorts removed. No further adjustment was needed. This method should work well with filters with many more crystals.

It is not surprising that adjustment should be difficult. Fig 10 shows the calculated response for this filter over a wider frequency range of 3.4 to 3.8 MHz. The nulls (calculated!) next to the desired passband are over 150 dB below the desired response! The "wings" that reappear at wider separations from the desired responses can be a problem. The crystal filter will have to be "protected" with a suitable L-C band-pass circuit; one or two resonators will probably do the job.

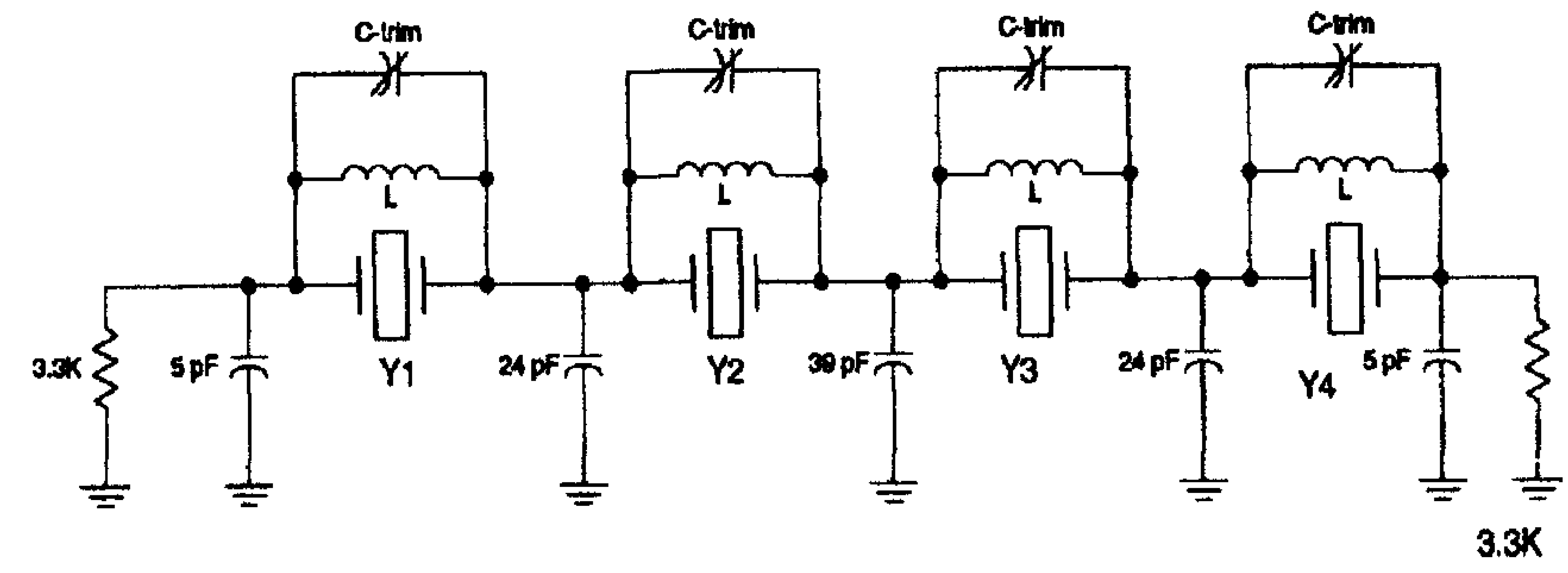



**Fig. 8—A low-frequency AM filter with BW = 3.5 kHz.**
**Y1,2,3,4—3.58-MHz surplus color-burst crystals. $L_m$ = 0.117 H, $C_o$ = 4 pF**
**L—151 µH, 48 turns #30 on FT-50-61 Ferrite toroid (Amidon).**
**C-trim—3-12 pF ceramic trimmer.**

## The G3UUR Method for Measuring Quartz Crystal Motional Parameters

This simple circuit may be used to measure the motional parameters of fundamental-mode quartz crystals. A crystal to be evaluated is placed in the circuit at Y1 and oscillation is confirmed. The frequency is measured. Then the switch is thrown and the frequency is measured again. Typical values are $C_p$ = 470 pF and $C_s$ = 33 pF. $C_m$ will have the same units as $C_s$:

If

$$C_s \ll C_p$$

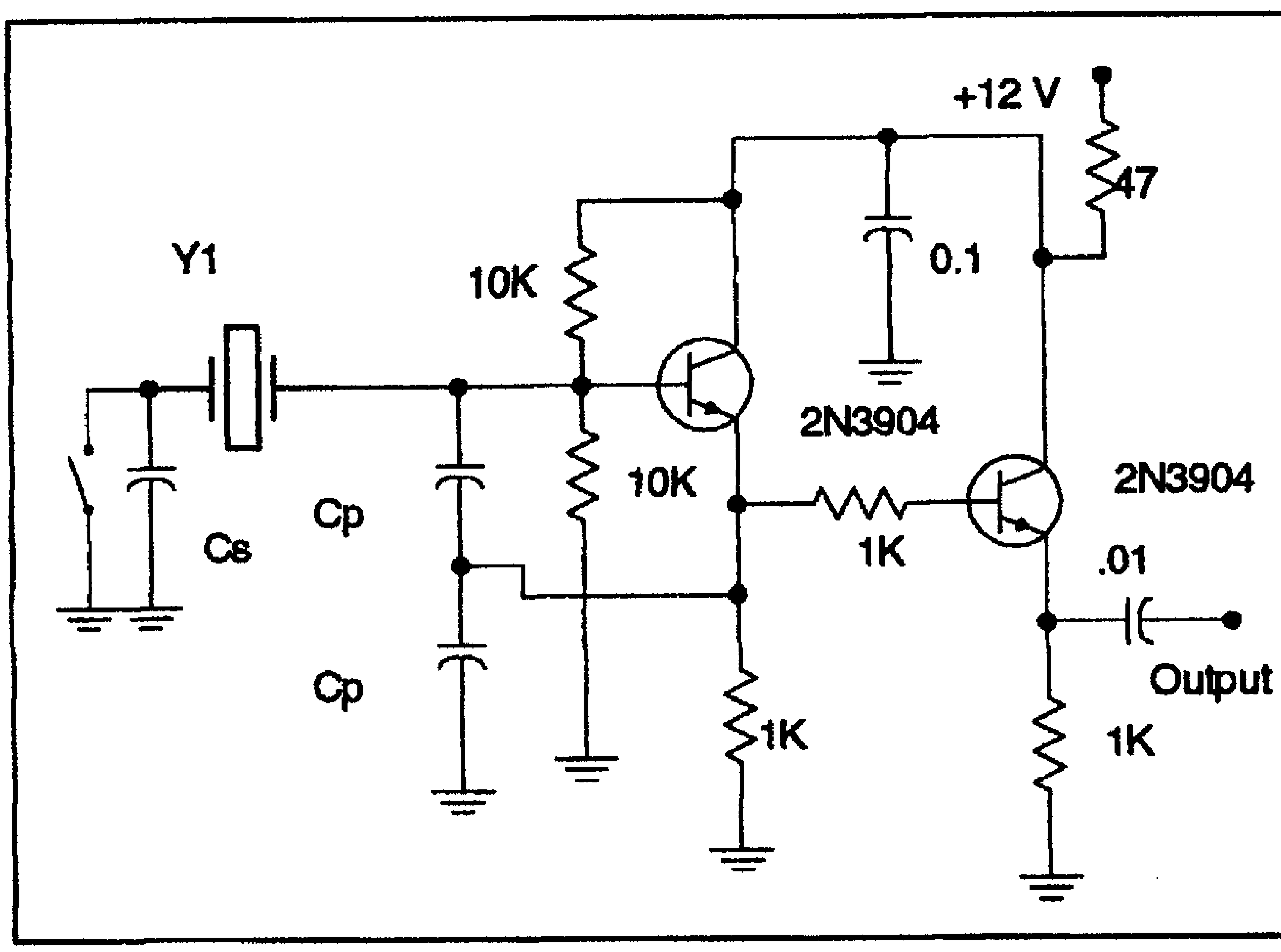
then

$$C_m \approx 2 \cdot C_s \cdot \frac{\Delta F}{F}$$

and

$$L_m = \frac{1}{\omega^2 \cdot C_m}$$

where $\omega = 2\pi F$ with $F$ in Hertz. $\Delta F$ is the frequency difference observed when the switch is activated. Example: use capacitors mentioned above, 10-MHz crystal; $F = 1 \times 10^7$, $\Delta F$ = 1609 Hz, to yield $L_m$ = 0.0239 H and $C_m$ = 10.6 fF. (1000 fF = 1 pF.)



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## Filters with Overtone Crystals

The example filter used readily available low-frequency crystals. The instrumentation used for measurement is that available to most experimentally active amateurs. Similar filters have been built at VHF with overtone crystals. Such filters are not difficult if small inductors are added to each crystal, along with small trimmers. The inductors can now use iron-powder cores. The difficulty with overtone crystals results from the physical nature of the crystals: while motional inductance remains essentially constant, independent of overtone number, the motional capacitance decreases in proportion to the square of the overtone number. Hence, a third-overtone crystal will have one ninth of the motional C that is seen at the fundamental frequency. The coupling capacitors will be reduced by a similar ratio, complicating filter realization. Swept instrumentation will probably be required for the construction of really good overtone filters.

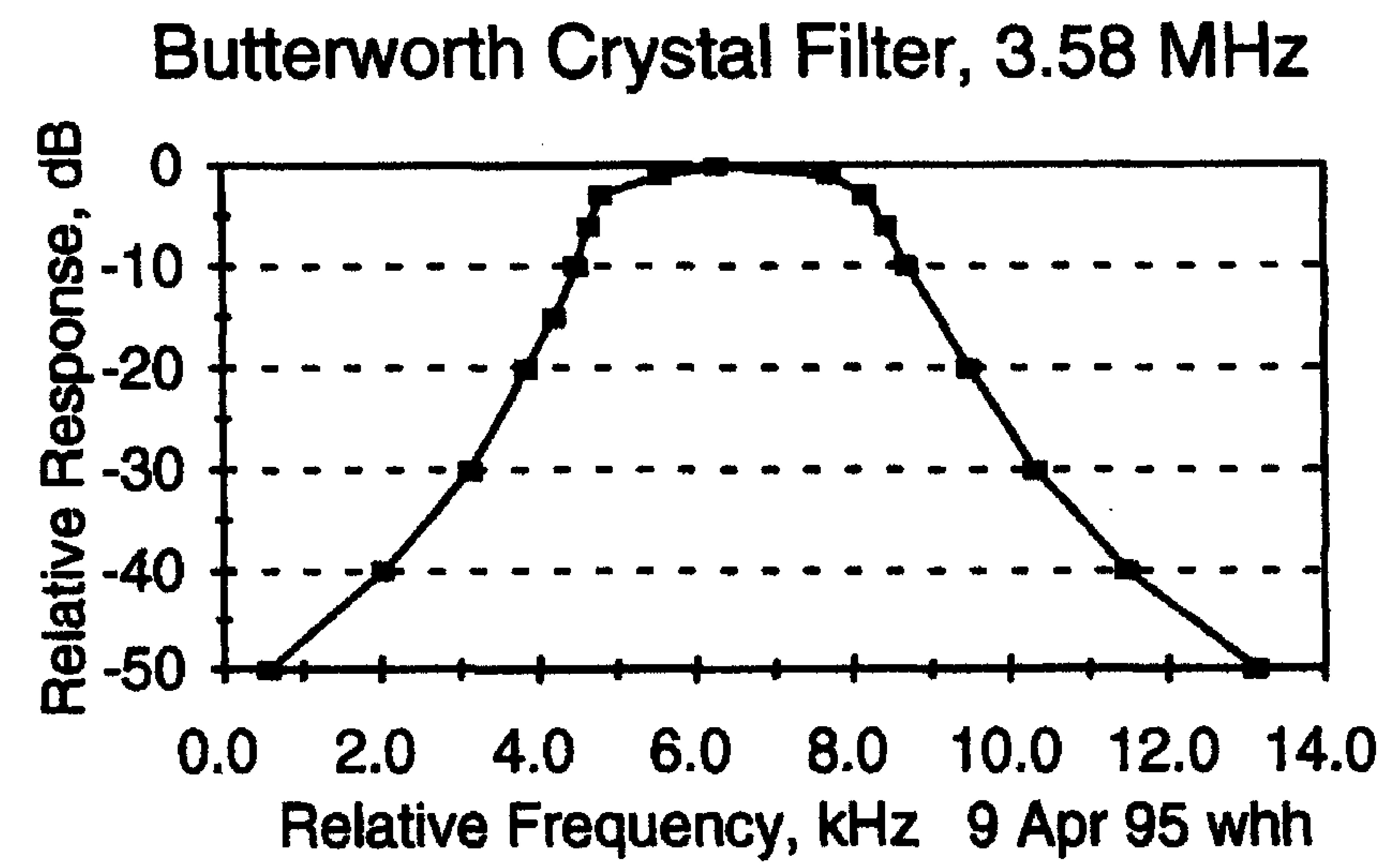


**Fig 9—Measured response of an AM-bandwidth crystal filter at 3.58 MHz.**

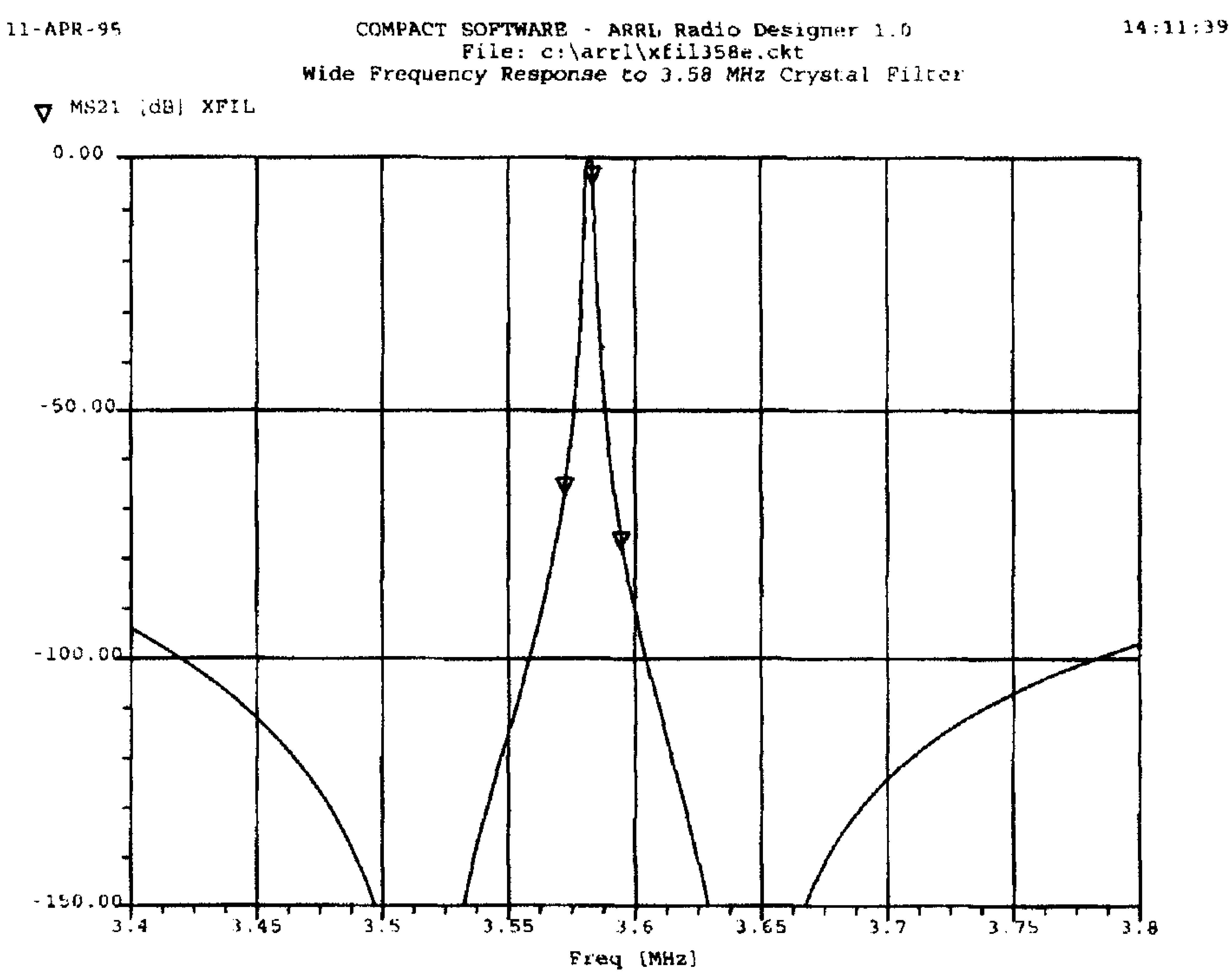


**Fig 10**

## Conclusions

The filters that we have examined are very practical. Of greater significance are the general methods that allow the experimenter to move toward high-order filters with improved selectivity and more constant group delay.

## Acknowledgments


Any long-term project like this one is the result of several sessions of midnight pondering and numerous discussions with others with similar interests. In this vein, I want to acknowledge discussions with Larry Lockwood, W7JBY, and Dr. Dave Gordon-Smith, G3UUR. It was Larry who first suggested that I examine reactance slope methods and Dr. Gordon-Smith who confirmed the approach through communications of his earlier works. More recent communications with Bill Carver, K6OLG/7, Jacob Makhinson, N6NWP, and Ulrich Rohde, KA2WEU, have provided the emphasis for the later work.


### Notes

[1]Carver, Bill, "High Performance Crystal Filter Design," *Communications Quarterly*, Winter 1993.

[2]Makhinson, Jacob, "Designing and Building High-Performance Crystal Ladder Filters," *QEX*, January 1995.

[3]Hayward, Wes, *Introduction to Radio Frequency Design*, ARRL 1994. See the crystal filter design programs on the disk distributed with the book including *X.EXE* and *MESHTUNE.EXE*.

[4]See Note 3, page 92.

[5]Zverev, *Handbook of Filter Synthesis*, John Wiley and Sons, Inc, 1967.

[6]Bottom, V. E., *Introduction to Quartz Crystal Unit Design*, Van Nostrand Reinhold Co, 1982.

[7]Gordon-Smith, Dave, G3UUR, private correspondence, June 26, 1982.